

Holographic description of confinement and screening through brane cosmology

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We compute the holographic quark potential in the realm of brane cosmology. We show that under certain conditions the very geometry due to an inflationary 3-brane induces a D3-D7-brane system. The cosmological constant that appears involved in the original geometry is attributed to the D7-brane position itself in its embedding process. We address the issues of confinement at low distances, screening effects at sufficiently large distances, and quark condensate.

I. INTRODUCTION

In order to extend the AdS/CFT correspondence [1] to accomplish QCD-like theories one needs to break supersymmetry and conformal invariance. The AdS/CFT correspondence is formulated around the background geometry of a stack of N D3-branes where the strings may end — for a review see [2–4]. The corresponding field theory is a gauge theory whose fields are in the adjoint representation of $SU(N)$ group. To take into account flavor degrees of freedom one needs to add N_f flavor branes. Now strings can be stretched between different types of branes with only one charge under the $SU(N)$ group on the D3-branes and then describe quark fields. The corresponding field theory now has fields (quarks) that are in the fundamental representation. One of the most interesting set up to accomplish this is the D3-D7-brane system with N_f D7 flavor branes in the probe limit ($N_f \ll N$) in order not to change the background geometry [5, 6]. Strings can have both ends on the flavor brane to describe a dual to quark-antiquark operators since they are in the adjoint representation of $SU(N_f)$ [2]. Such effort is one among several others in the direction to achieve QCD-like theories.

In a general perspective one can think of this proposal as a way to ‘deform’ the AdS space. For instance, for confining theories see [7–13]. In [14] was assumed a deviation from the conformal case which is *naturally* obtained in geometries of brane cosmology scenarios [15, 16]. As we shall see, a small deviation around AdS space controlled by the tension and the cosmological constant on the brane is encoded on a fundamental constant C . It was found that such a deviation that leads

to a *confining regime* in the dual gauge field theory is directly related to cosmological constant on the brane. Indeed, this encloses a general class of geometries where the brane has a cosmological constant, i.e., a *bent* brane, such as AdS_4 or dS_4 branes [17–24], in contrast with flat D3-branes in the original AdS/CFT correspondence. Other inflationary brane scenarios can also be found in [21, 25–28].

In this paper we revisit the previous results in [14] to address the regimes of confinement and screening from the perspective of D3-D7-brane system. As we shall show, by making suitable transformation of coordinate and embeddings the very geometry due to an inflationary 3-brane [15, 16] induces a D3-D7-brane system, in Euclidean signature. For further details on Euclidean brane cosmology see [29, 30]. The D7-brane position will be directly related to the constant C . In addition to the several reasons raised above to choose such a system, we recall that from the point of view of cosmological selection of the number of spatial dimensions, a ten-dimensional universe initially filled with a dilute Dp -anti- Dp -brane gas tends to find its equilibrium filled with only D3 and D7-branes in the realm of type IIB superstrings [31].

The paper is organized as follows. In Sec. II we explore the emergence of an induced D7-brane through brane cosmology. In Sec. III we address the issue of the D7-brane embedding and show that the quark condensate is a linear function of the quark mass. It should be stressed that the flavoring is due to the presence of the cosmological constant. In Sec. IV, we compute the interquark potentials from the holographic (‘AdS/QCD’) point of view [1, 7–9, 32–40] by relating the string world-sheet area along the bulk with the area of the Wilson loop on the boundary in Euclidean signature. Finally, in Sec. V we present our final discussions.

II. BRANE COSMOLOGY AND THE EMERGENCE OF INDUCED D7-BRANE

We shall mainly focus on the fact that the brane cosmology metric [15, 16] (in its Euclidean form) as has been shown in [14] can be written as follows

$$ds_5^2 = \alpha' \left[\frac{U_0^2}{R^2} y^2 (dt^2 + a_0^2 \gamma_{ij} dx^i dx^j) + \frac{R^2}{y^2 - C} dy^2 \right], \quad (1)$$

where

$$C = \frac{2\gamma R^2}{\alpha' U_0^2}, \quad \gamma = 3H_E^2/2\sigma, \quad \gamma_{ij} = \delta_{ij}, \quad i, j = 1, 2, 3 \quad (2)$$

being H_E the Euclidean Hubble parameter defined in terms of an induced Friedmann equation on the inflating 3-brane with tension σ [14–16] that we identify with the D3-brane tension τ_3 [14]. We

also assume flat universe ($k = 0$) thus $\gamma_{ij} = \delta_{ij}$. Notice that the coordinate $U = U_0 y$ is assumed to have dimension of energy, such that $U_0^2 \rightarrow U_0^2/\alpha'$ and $R^4 \equiv 4\pi g_s N \alpha'^2 \rightarrow R^4 \equiv 4\pi g_s N$. We should note that this metric can be rewritten in a suitable form by *changing the variables* $\rho^2 = U_0^2 y^2 - \tilde{C}$ and $\tilde{C} = U_0^2 C$ such that we find

$$ds_5^2 = \alpha' \left[\frac{\rho^2 + \tilde{C}}{R^2} (dt^2 + a_0^2 \gamma_{ij} dx^i dx^j) + \frac{R^2}{\rho^2 + \tilde{C}} d\rho^2 \right] \quad (3)$$

This metric can be seen as the induced metric on a D7-brane if we want our setup to be developed by a geometry in a ten-dimensional spacetime generated by a stack of N D3-brane and a number of probe D7-branes (N_f D7 flavor branes) [5, 6] where the strings can end. Since we are considering probe D7-branes ($N_f \ll N$), there is a way to turn the original metric to an induced D7-brane metric by keeping it as a solution of the original equations of motion. Thus, let us first complete the metric (3) by extending the number of dimensions involved in the ‘radial’ coordinate as follows

$$\rho^2 = X_4^2 + X_5^2 + X_6^2 + X_7^2 \quad d\rho^2 \rightarrow d\rho^2 + \rho^2 d\Omega_3^2 = dX_4^2 + dX_5^2 + dX_6^2 + dX_7^2 \quad (4)$$

such that we have the induced metric on a D7-brane

$$ds_8^2 = \alpha' \left[\frac{\rho^2 + L^2}{R^2} (dt^2 + a_0^2 \gamma_{ij} dx^i dx^j) + \frac{R^2}{\rho^2 + L^2} (d\rho^2 + \rho^2 d\Omega_3^2) \right] \quad (5)$$

where $L \equiv \sqrt{\tilde{C}} = U_0 \sqrt{C}$. One should notice that for $\rho \rightarrow 0$ (which means that $U_0^2 y^2 \sim L^2$, i.e., near the place where the D7 brane sits in the 89-plane — see below) the radius of S^3 shrinks to zero and the ds_8^2 metric approaches the original ds_5^2 metric. As we shall see later, L is the place where the D7-brane sits and will be directly related to the quark mass m_q . Notice that for $\rho \gg L$ (or for $L = 0$) the metric approaches a geometry with the topology $AdS_5 \times S_3$ which simple reflects the absence of dynamical quarks with light masses m_q and the strings turn out to be ending on the AdS boundary generating quarks with infinite mass. In a general situation the strings end on the D7-branes generating quarks with light masses presenting both confinement and flavoring.

III. FLAVORING WITH THE PRESENCE OF THE COSMOLOGICAL CONSTANT

The embedding of the D7-brane. To address the issue of D7-brane embedding one should consider the complete metric in a ten-dimensional spacetime

$$ds_{10}^2 = \alpha' \left[\frac{\rho^2 + X_8^2 + X_9^2}{R^2} (dt^2 + a_0^2 \gamma_{ij} dx^i dx^j) + \frac{R^2}{\rho^2 + X_8^2 + X_9^2} (d\rho^2 + \rho^2 d\Omega_3^2 + dX_8^2 + dX_9^2) \right] \quad (6)$$

that by using symmetry in the $X_8 - X_9$ plane and setting $X_9 = 0$ and $X_8 = \omega(\rho)$ we find the induced metric for the D7-brane

$$ds_8^2 = \alpha' \left[\frac{\rho^2 + \omega(\rho)^2}{R^2} (dt^2 + a_0^2 \gamma_{ij} dx^i dx^j) + \frac{R^2}{\rho^2 + \omega(\rho)^2} ((1 + (\partial_\rho \omega)^2) d\rho^2 + \rho^2 d\Omega_3^2) \right] \quad (7)$$

The embeddings $\omega(\rho)$ is obtained by solving the equation of motion deduced from the D7-brane action given up to angular factors by

$$S_{D7} = -\tau_7 \int d^8 \xi a_0^3 \rho^3 \sqrt{1 + \omega'(\rho)^2}, \quad (8)$$

where $\omega'(\rho) \equiv \partial_\rho \omega$ and the D7-brane tension $\tau_7 = [(2\pi)^7 g_s \alpha'^4]^{-1}$. We have used the metric (7) and disregarded the gauge potentials on the D7-brane world-volume. The equation of motion

$$\frac{d}{d\rho} \left[\frac{\rho^3}{\sqrt{1 + \omega'(\rho)^2}} \frac{d\omega}{d\rho} \right] = 0 \quad (9)$$

has the well-known asymptotic solution given by

$$\omega(\rho) \sim L + \frac{c}{\rho^2}, \quad \rho \rightarrow \infty \quad (10)$$

where c corresponds to the v.e.v. of an operator that has dimension three since ρ carries energy dimension and measures the quark condensate $\langle \bar{\psi} \psi \rangle$. L is an operator with energy dimension previously defined as

$$L = U_0 \sqrt{C} = \frac{\sqrt{2} \sqrt{\gamma} R}{\sqrt{\alpha'}} \quad (11)$$

where we have used the explicit form of C given in (2). Now considering $L \equiv 2\pi m_q$ allows us to define the quark mass m_q . It can be written explicitly as follows

$$m_q = \frac{1}{\pi \sqrt{2}} \sqrt{\gamma} \frac{R}{\sqrt{\alpha'}}. \quad (12)$$

Here instead of solving numerically Eq. (9) to find the embeddings we take advantage of using the asymptotic relationship between $\omega(\rho)$ and U previously stated. Thus, now starting with the original cosmological solution [14] we can see that

$$U^2(r) = \gamma + \xi^2 e^{\mu r} + \chi^2 e^{-\mu r} \rightarrow U^2(r) = \xi^2 e^{\mu r} \equiv \rho^2 \quad \text{as} \quad r \rightarrow \infty \quad (13)$$

that solving for $\exp(\mu r)$ we find asymptotically

$$U^2(r) = \gamma + \rho^2 + \frac{\xi^2 \chi^2}{\rho^2}, \quad \rho \rightarrow \infty \quad (14)$$

Now restoring the dimension of energy ($U^2 \rightarrow U^2/\alpha'$, with dimensionless R), recognizing that $\omega^2(\rho) = U^2 - \rho^2 \sim L^2$ asymptotically (which is consistent with change of variables in the metric (3)) and using the fact that $\xi^2\chi^2 = \frac{\gamma^2}{4}$, which is restricted to the region $\gamma \leq 1/2$ [14] we have

$$\omega^2(\rho) = \frac{\gamma}{\alpha'} + \frac{\gamma^2}{4\alpha'^2\rho^2} \rightarrow \omega(\rho) \simeq \sqrt{\frac{\gamma}{\alpha'}} \left(1 + \frac{\gamma}{8\alpha'\rho^2}\right) \quad (15)$$

Comparing with Eq. (12) we can rewrite Eq. (15) as

$$\omega(\rho) \simeq \frac{2\pi m_q}{\sqrt{2}R} + \frac{(2\pi m_q)^3}{8(\sqrt{2}R)^3\rho^2}. \quad (16)$$

Notice that this formula is dimensionally consistent with the asymptotic solution Eq. (10). Thus, up to a common $\sqrt{2}R$ factor, we simply find

$$\begin{aligned} \omega(\rho) &\simeq 2\pi m_q + \frac{2\pi^2 m_q^2}{R^2} \frac{2\pi m_q}{8\rho^2} \\ &\simeq 2\pi m_q + \frac{2\pi m_q M^2}{8\rho^2}, \quad \text{where} \quad M^2 = \frac{2\pi^2 m_q^2}{R^2} = \pi\sigma_c \end{aligned} \quad (17)$$

As we shall show later, σ_c is the tension of the confining ‘QCD’ string. The condensate operator can be now identified with

$$c = \frac{1}{4}\pi m_q M^2. \quad (18)$$

For fixed σ_c , the condensate increases *linearly* with the quark mass m_q [41–43]. This seems to be natural for light m_q masses [2]. Notice that for $M = m_q = m$ this relationship between quark condensate and quark mass has an interesting resemblance to *Ioffe formula* [44] for nucleon masses

$$m \approx (4\pi^2 \langle \bar{\psi}\psi \rangle)^{1/3}. \quad (19)$$

IV. THE INTERQUARK POTENTIAL

In order to follow with our objective such as to obtain the energy and inter-quark distances we take the relevant part of the AdS_5 metric of the background (6)

$$ds_5^2 = \alpha' \left[\frac{X_4^2}{R^2} (dt^2 + a_0^2 \gamma_{ij} dx^i dx^j) + \frac{R^2}{X_4^2} dX_4^2 \right] \rightarrow \alpha' \left[\frac{U_0^2}{R^2} y^2 (dt^2 + a_0^2 \delta_{ij} dx^i dx^j) + \frac{R^2}{y^2} dy^2 \right], \quad (20)$$

This geometry can be identified with a near horizon geometry of a stack of N D3-branes located at $X_4 = X_5 = \dots = X_9 = 0$ and should not be modified by the N_f D7-branes as long as one assumes the probe limit $N_f \ll N$.

The distance between quarks is given by [35]

$$\tilde{L} = 2 \int_{r_0}^{r_1} dr \frac{g(\tau, r)}{f(\tau, r)} \frac{f(\tau, r_0)}{\sqrt{f^2(\tau, r) - f^2(\tau, r_0)}} = \frac{L_0}{a_0(t)} \quad (21)$$

where using the original metric given in brane cosmology [14-16] $g^2(\tau, r) = g_{00}g_{rr} = U^2$ and $f^2(\tau, r) = g_{00}g_{xx} = U^4 a_0^2$ we find

$$L_0 = 2 \int_{r_0}^{r_1} dr \frac{1}{U} \frac{U_0^2}{\sqrt{U^4 - U_0^4}} \quad (22)$$

which is in general the ‘co-moving inter-quark distance’. Finally using the coordinates adopted in the metric (20) we find the useful formula

$$L_0 = 2 \frac{R^2}{U_0} \int_1^{U_1/U_0} \frac{dy}{y^2 \sqrt{y^4 - 1}} \quad (23)$$

Similarly we have the Nambu-Goto action [14, 35]

$$S = 2 \frac{1}{2\pi\alpha'} \int_{r_0}^{r_1} \int_0^T dr d\tau \frac{g(\tau, r)f(\tau, r)}{\sqrt{f^2(\tau, r) - f^2(\tau, r_0)}} = 2 \frac{T}{2\pi\alpha'} \int_{r_0}^{r_1} dr \frac{U^3}{\sqrt{U^4 - U_0^4}}.$$

Again using the coordinates adopted in the metric (20) and the expectation value of the Wilson loop $\langle W(C) \rangle \sim e^{-E(L_0)T} \sim e^{-S}$ where S is the Nambu-Goto action we find the useful formula for the inter-quark energy

$$E = 2 \frac{U_0}{2\pi} \int_1^{U_1/U_0} \frac{y^2}{\sqrt{y^4 - 1}} dy. \quad (24)$$

The equations (23) and (24) are fundamental to obtain the inter-quark potentials and to study several regimes. In both cases we shall assume the endpoints $U_1 = L = 2\pi m_q = U_0 \sqrt{C}$ of the string symmetric profile ending where the D7-brane sits. Thus, we are understanding the upper limit as $U_1/U_0 \equiv \sqrt{C}$. In the following we are going to consider large and small separation between the quarks, as compared to the scale of the lightest meson.

A. Large interquark distance

In this case we should consider $U_1/U_0 \equiv \sqrt{C} \gg 1$ [3]. Firstly for L_0 we rewrite Eq. (23) as follows

$$\begin{aligned} \frac{L_0}{2} &= \frac{R^2}{U_0} \int_1^{U_1/U_0} \frac{dy}{y^2 \sqrt{y^4 - 1}} = \frac{R^2}{U_0} \int_1^\infty \frac{dy}{y^2 \sqrt{y^4 - 1}} - \frac{R^2}{U_0} \int_{U_1/U_0}^\infty \frac{dy}{y^2 \sqrt{y^4 - 1}} \\ &\simeq \frac{R^2}{U_0} \kappa - \frac{R^2}{U_0} \int_{U_1/U_0}^\infty \frac{dy}{y^4} \\ &\simeq \frac{R^2}{U_0} \kappa - \frac{1}{3} \frac{R^2 U_0^2}{U_1^3} \end{aligned} \quad (25)$$

In the above integrations at r.h.s., the first term gets $\kappa = \frac{1}{\sqrt{2\pi}}\Gamma\left(\frac{3}{4}\right)^2$ and the integrand in the second term is well approximated by y^{-4} since $y \gg 1$ in that interval. Now recalling that $U_1 = 2\pi m_q$ we find

$$\frac{L_0}{2} \simeq \frac{R^2}{U_0} \kappa - \frac{1}{3} \frac{R^2 U_0^2}{(2\pi m_q)^3} \quad (26)$$

Let us now solve this equation iteratively for U_0 to obtain

$$U_0 \simeq \frac{2R^2 \kappa}{L_0} \left[1 - \frac{1}{3} \frac{R^6 \kappa^2}{(\pi L_0 m_q)^3} \right] \quad (27)$$

It is interesting to note that taking the leading term we find

$$\frac{m_q}{U_0} \simeq \frac{m_q L_0}{2R^2 \kappa} \sim L_0 m_{\text{gap}}, \quad (28)$$

where $m_{\text{gap}} \sim m_q / \sqrt{g_s N}$. This implies that our approximation is valid as long as $L_0 m_{\text{gap}} \rightarrow \infty$.

Now for energy we rewrite Eq. (24) as follows

$$E = \frac{U_0}{\pi} \int_1^{U_1/U_0} \frac{y^2}{\sqrt{y^4 - 1}} dy = \frac{U_0}{\pi} \int_1^\infty \frac{y^2}{\sqrt{y^4 - 1}} dy - \frac{U_0}{\pi} \int_{U_1/U_0}^\infty \frac{y^2}{\sqrt{y^4 - 1}} dy, \quad (29)$$

that regularizing each integral in the r.h.s. by properly subtracting the diverging terms

$$\int_0^\infty dy = \int_1^\infty dy + \int_0^1 dy, \quad \int_0^\infty dy = \int_{U_1/U_0}^\infty dy + \int_0^{U_1/U_0} dy \quad (30)$$

we find

$$E = 2m_q + \frac{U_0}{\pi} \left[\int_1^\infty \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) dy - 1 \right] - \frac{U_0}{\pi} \int_{U_1/U_0}^\infty \left(\frac{y^2}{\sqrt{y^4 - 1}} - 1 \right) dy. \quad (31)$$

The first integral can be precisely given in terms of $-\kappa$ and the second integral due to the large interval can be computed by considering the integrand well approximated by $(1/2)y^{-4}$. Thus we can find the energy given in the form

$$E \simeq 2m_q - \frac{U_0 \kappa}{\pi} \left(1 - \frac{1}{6} \frac{U_0^3}{\kappa U_1^3} \right). \quad (32)$$

Now substituting $U_0 \simeq 2R^2 \kappa / L_0$, i.e., the leading term of Eq. (27), and $U_1 = 2\pi m_q$ into Eq. (32) we find

$$E \simeq 2m_q - \frac{\alpha^2}{2L_0} \left(1 - \frac{1}{6} \frac{R^6 \kappa^2}{(\pi L_0 m_q)^3} \right), \quad (33)$$

where $\frac{\alpha^2}{2} = \frac{2R^2 \kappa^2}{\pi}$. Notice that light quarks here stand for a sufficiently small cosmological constant present in γ — See Eq. (12). This confirms the experimental results pointing the hadronization of light mesons.

B. Small interquark distance

In the present case we should consider $U_1/U_0 \equiv \sqrt{C} \rightarrow 1$ [3]. This is the same condition previously considered in Ref. [14] do address the confining regime. Again, firstly for L_0 we rewrite Eq. (23) as follows

$$\begin{aligned} \frac{L_0}{2} &= \frac{R^2}{U_0} \int_1^{U_1/U_0 \rightarrow 1} \frac{dy}{y^2 \sqrt{y^4 - 1}} \simeq \frac{R^2}{U_0} \int_1^{U_1/U_0 - 1} \frac{dz}{(1+2z)\sqrt{(1+4z)-1}} \\ &\simeq \frac{R^2}{U_0} \int_1^{U_1/U_0 - 1} \frac{dz}{2\sqrt{z}} \\ &\simeq \frac{R^2}{U_0} \sqrt{\frac{U_1}{U_0} - 1} \end{aligned} \quad (34)$$

where in the r.h.s. we have considered $y = 1 + z$ and $z \ll 1$ in the short interval above. Let us write (34) in terms of U_0 , that is

$$U_0 \simeq U_1 \left[1 + \frac{U_0^2}{4R^4} L_0^2 \right]^{-1} \simeq U_1 \left[1 - \frac{U_0^2}{4R^4} L_0^2 \right] \quad (35)$$

Solving iteratively for U_0 recalling that $U_0 \simeq U_1$ and $U_1 = 2\pi m_q$ we find

$$U_0 \simeq 2\pi m_q \left[1 - \left(\frac{\pi L_0 m_q}{R^2} \right)^2 \right]. \quad (36)$$

Now for energy we rewrite Eq. (24) as follows

$$\begin{aligned} E &= \frac{U_0}{\pi} \int_1^{U_1/U_0 \rightarrow 1} \frac{y^2}{\sqrt{y^4 - 1}} dy \simeq \frac{U_0}{\pi} \int_1^{U_1/U_0 - 1} \frac{1+2z}{\sqrt{(1+4z)-1}} dz \\ &\simeq \frac{U_0}{2\pi} \int_1^{U_1/U_0 - 1} z^{-1/2} dz \\ &\simeq \frac{U_0}{\pi} \sqrt{\frac{U_1}{U_0} - 1} \end{aligned} \quad (37)$$

Again, in the r.h.s. we have considered $y = 1 + z$ and $z \ll 1$ in the short interval above. Substituting (36) into (37) and recalling that $U_1 = 2\pi m_q$ the energy is now given by the linear form

$$E \simeq \frac{2\pi m_q^2}{R^2} L_0 \rightarrow E = \sigma_c L_0, \quad \sigma_c = \frac{\gamma}{\pi \alpha'} \quad (38)$$

where in the last step we have used the mass formula (12) — notice the relationship between σ_c and the mass scale M defined in Eq. (17). It gives the same result obtained in Ref. [14]. This means a confining regime at sufficiently small interquark distance [3].

V. CONCLUSIONS

The cosmological constant that appears involved in the original geometry is attributed to the D7-brane position itself in its process of embedding. As a consequence we find an explicit dependence of the quark mass with the cosmological constant. Analogously, direct computation with the original geometry gives the same interquark potential regimes found in Ref. [14]. Thus, our results show that we have found a complete equivalence of flavoring with a D7-brane or properly considering a cosmological solution. We also have found an explicit linear dependence of the quark condensate with the quark mass. This seems to be the case for small masses, in the region where the quenching $1/m$ [41] still does not take place. So further investigations related to the quark condensate, for instance, in the present setup should be pursued.

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